Mode-Locked Oscillator Arrays

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Abstract—**The output characteristics of a linear array of oscillators is discussed. By arranging the output frequencies of each oscillator to give a spectrum of equally spaced components, a "mode-locked" system results, thereby producing short, high-power RF pulses. Promising results from a linear array of up to five elements is presented, with good theoretical agreement.**

I. Background

MODE-LOCKING refers to a situation in which a system is made to oscillate in a number of equally spaced spectral modes, with comparable amplitudes and with locked phases. This technique has been used extensively in lasers to produce high-power optical pulses with durations of tens of femtoseconds [1]. In such a system the peak output power is N^2 times the average output power of a single oscillation mode, where N is the number of modes locked, and the repitition rate of the pulses corresponds to the frequency spacing of the modes.

Recent developments in quasi-optical oscillating arrays [2], [3] appear to have opened the door to a range of new laser-like effects at microwave and millimeter frequencies. One such array [2] uses planar active radiating elements that interact weakly with neighboring elements through mutual coupling mechanisms. To date this work has concentrated on synchronizing the array elements to the same frequency for coherent power-combining. However the same arrays can be used as a mode-locked system by adjusting the individual oscillation frequencies to produce a comb spectrum, with equally spaced components as shown in Fig. 1(a). Even for a relatively small array of five elements, mode-locking results in peak powers that are approximately 25 times the power obtained from a single element. Furthermore, high-pulse repetition rates can be produced quite easily.

The mode-locking phenomenon can be easily understood by representing the oscillation modes as a set of rotating phasors in the complex plane, as shown in Fig. 1(b). It is assumed that there are $N = 2n + 1$ modes with frequencies

$$\omega = \omega_0 - l\Delta\omega, \qquad l = -n, \cdots, n, \tag{1}$$

where l is the mode index, and the phases are locked such that

$$\phi_l - \phi_{l-1} = \phi, \qquad l = -n + 1, \cdots, n, \tag{2}$$

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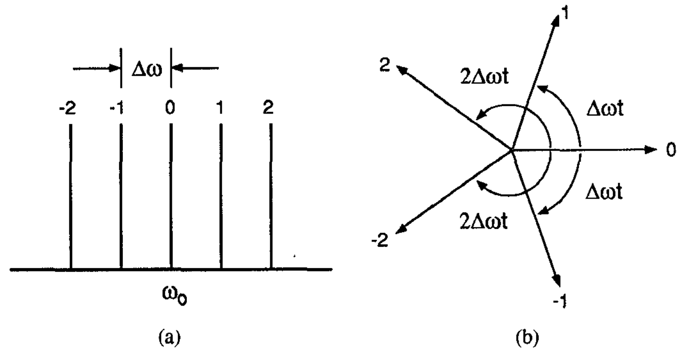

Fig. 1. (a) Frequency spectrum of a 5-element mode-locked oscillator array. (b) Phasor description in the complex plane at one instant of time.

and that all modes have the same amplitude E_0. The phasors rotate at an angular frequency $\omega = \omega_0 - l\Delta\omega$. If we move to a frame of reference rotating at ω_0 then the $l = 0$ mode will be stationary and the other phasors will rotate at $l\Delta\omega$, which will be clockwise or counter-clockwise depending on the sign of l. At time $t' = 0$ all the phasors line up and the signal will be a maximum. The phasors will again line-up at time $t' = 2\pi/\Delta\omega$ (Fig. 1(b)). Mathematically, the total electric field can be written as

$$\begin{aligned} E(t) &= \sum_{l=-n}^{n} E_0 \exp\{j[(\omega_0 - l\Delta\omega)t + l\phi]\} \\ &= A(t)\exp(j\omega_0 t), \end{aligned} \tag{3}$$

where $A(t)$ is given by

$$A(t') = E_0 \frac{\sin[(2n+1)\Delta\omega t'/2]}{\sin[\Delta\omega t'/2]}, \tag{4}$$

and $\Delta\omega t' = \Delta\omega t + \phi$ [1]. The output signal can be viewed as an amplitude modulated carrier at ω_0, where $A(t)$ is the modulation. The maximum amplitude at $t' = 0$ is given by

$$\lim_{t' \to 0} E_0 \frac{\sin[(2n+1)\Delta\omega t'/2]}{\sin[\Delta\omega t'/2]} = E_0(2n+1), \tag{5}$$

corresponding to a peak power of $E_0^2(2n+1)^2$. If the elements were synchronized to the same frequency (as in a power-combining array), the average output power would be $E_0^2(2n+1)$.

In laser applications there are a number of different approaches for obtaining "locked" phases according to (2). These approaches are broadly characterized as either active mode-locking, where an external source modulates some

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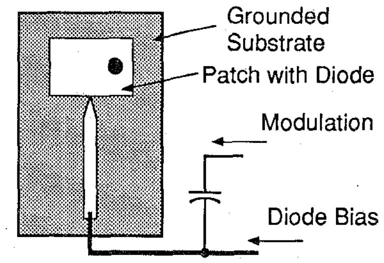

Fig. 2. Active radiating element, using a patch antenna and Gunn diode. Bias is applied at an RF null on the patch, and modulation signals are capacitively coupled to the bias line.

parameter in the laser, or passive mode-locking such as the case when a saturable absorber is used in the laser cavity. In the case of active mode-locking the external modulation can act to AM mode-lock the system by varying the loss in the cavity, of FM mode-lock the laser by varying the optical length of the cavity. Analogous ideas may be applied to a quasi-optical array of millimeter/microwave oscillators. However the analogy can only be carried so far.

If we modulate an element of the array, sidebands are generated which can than be used to injection-lock neighboring array elements [4]. This process can be repeated to produce an array of mode-locked oscillators. The phenomena differs from the optical case because the coupling between oscillators tends to pull the oscillators. If the mode spacing is less than the locking bandwidth the oscillators will tend to lock to a single frequency. However, if the spacing exceeds the locking bandwidth, this frequency pulling generates a comb spectrum [4], which also enables mode-locking of the array.

II. Oscillator Arrays

The oscillator arrays under consideration are exactly the same as those used in previous work on power-combining [2], except that only small linear arrays were used here for simplicity, and the synchronous mode of operation is avoided. Each element of the array is a self-contained oscillator, consisting of a radiating element and active device. A typical configuration is shown in Fig. 2, and uses a Gunn diode and patch antenna [5].

In this work, commercially available MA/COM packaged Gunn diodes (MA49104) were used. Each element measured 9.5 mm × 8.0 mm, with the diode located 2.0 mm from the edge, and fabricated on 60 mil, $\varepsilon_r = 4.1$ substrate. Typical bias of 10 V gave 11 GHz oscillations. Bias was applied at a low impedance point on a nonradiating edge. The elements can be frequency modulated for active mode-locking by capacitively coupling a modulation signal onto the bias line, as shown in the figure.

Mutual pulling effects between the oscillators arise due to radiative coupling mechanisms. This coupling can be controlled by the element spacing, or by using a quasi-optical reflector above the array [1]. This reflector forms a Fabry–Perot cavity with the oscillator array. Since the oscillators have typically low Q-factors of 20–30, the external cavity can also help to stabilize the array [6].

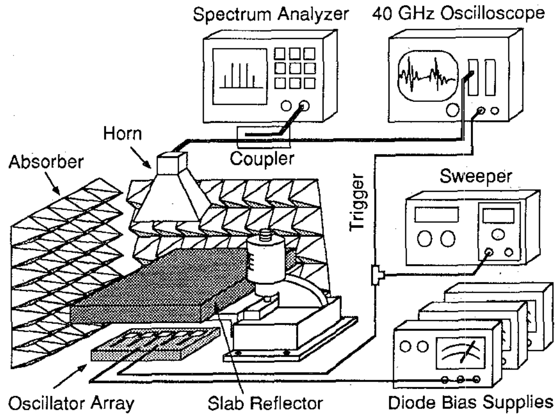

Fig. 3. Measurement setup for RF pulse generation and detection. Output from the coupled oscillator array is observed in both the time- and frequency-domain. Sweeper provides a modulation signal to the array, which is also used to trigger the oscilloscope. Dielectric slab reflector is also shown, which helps stabilize the array.

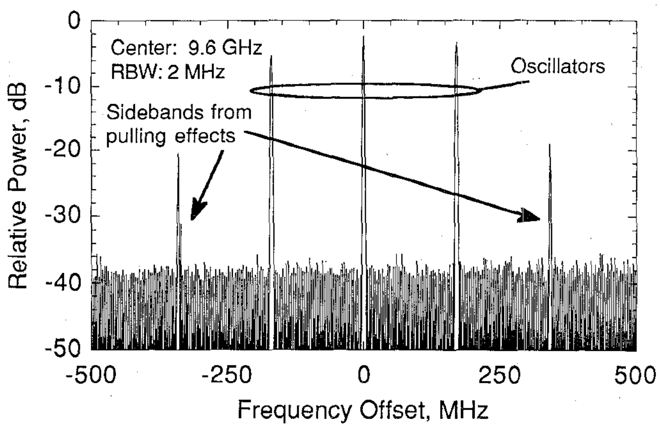

Fig. 4. Spectrum of three mode-locked microwave oscillators. Additional sidebands arise from frequency pulling effects.

III. Measurements

The experimental setup is illustrated in Fig. 3. Self mode-locking of the oscillators was established by first adjusting the oscillation frequencies of each element (using the bias supplies) to give equally spaced spectral components. This can be done easily using a spectrum analyzer; the spectrum of three oscillators, made using an HP 8562A spectrum analyzer is shown in Fig. 4. Since the elements pull each other, this operation must be performed with all elements operating simultaneously. Note from Fig. 4 that additional sidebands are produced as a result of the mutual pulling effects (the spectrum of a pulled oscillator has been described elsewhere [2]). As mentioned previously, the pulse repetition rate is inversely proportional to the frequency difference, which was constrained by the locking range of the oscillators to 50 MHz or more.

With the frequencies set as described, a periodic train of pulses was observed on a Tektronix CSA803 communications signal analyzer. The analyzer was equipped with an SD-24 TDR/sampling head with an equivalent bandwidth of 20 GHz. For the self-mode-locking operation, a triggering signal was derived by using a low-pass filter to remove the 11

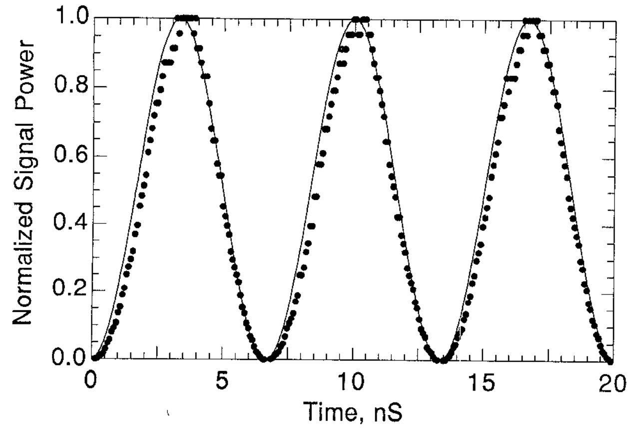

Fig. 5. • Measured and—theoretical time dependence of the output signal power envelope of two mode-locked oscillators.

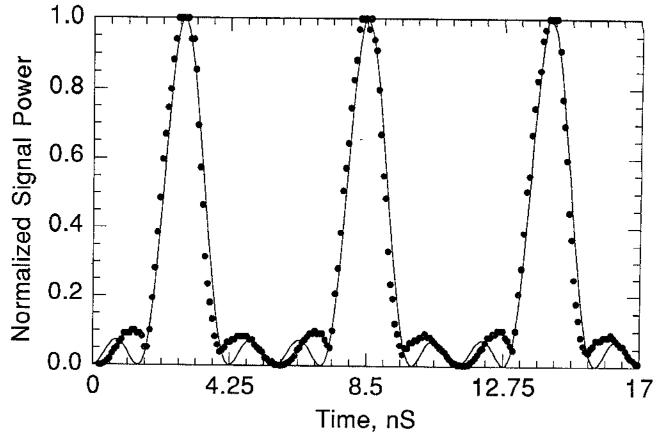

Fig. 7. • Measured and—theoretical time dependence of the output signal power envelope of four mode-locked oscillators.

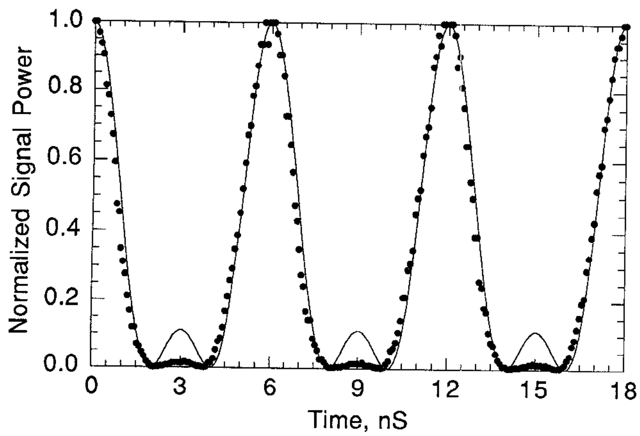

Fig. 6. • Measured and—theoretical time dependence of the output signal power envelope of three mode-locked oscillators.

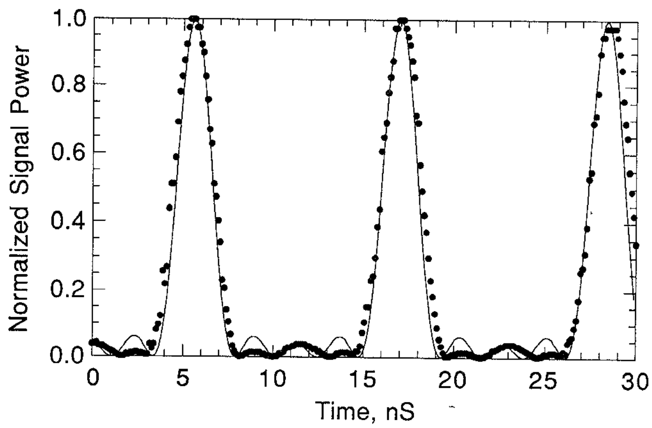

Fig. 8. • Measured and—theoretical time dependence of the output signal power envelope of five mode-locked oscillators.

GHz carrier. In this way, the envelope of the carrier could be observed quite easily. In addition, the CSA803 provides an envelope detection feature, which was used to present the measurements that follow.

More stable and reproducible waveforms were observed using an "active" mode-locking scheme, in which one or more of the elements were frequency modulated at the frequency difference $\Delta\omega$. The modulation signal was provided by an RF sweeper, which was also used to provide a triggering signal for the oscilloscope. The addition of a reflector element above the array was also found to improve the stability of operation. Mode-locking under these conditions was verified for two, three, four, and five oscillators, as shown in Figs. 5–8. Theoretical curves based on the assumption of equal amplitudes are also shown for comparison; even better agreement could be obtained by accounting for the slightly nonuniform oscillator amplitudes and the additional sidebands. The theory curves are given by $(A(t)/E_0)^2$, where $A(t)$ is given by (4). Note that the ordinate axes in each figure is the signal power, not amplitude.

IV. Conclusion

The results presented suggest that it may be possible to generate high-power RF pulses with high-pulse repetition frequencies, using planar arrays of devices. This quasi-optical array architecture is expected to scale up to the Terahertz frequency range using suitable devices.

However there are certain difficulties with the previous scheme that need to be addressed before large arrays can be operated. In a mode-locked laser each oscillating mode represents a cavity resonance, and hence the spectrum inherently contains equally spaced frequency components. In the present situation, each "mode" is the output of a free-running oscillator. We have chosen to enforce the desired spectrum by manually adjusting the oscillator frequencies, but clearly this would not be feasible in a large array. Some additional mechanism for encouraging the mode-locked behavior could be useful, such as the nonlinear element in a passively mode-locked laser.

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